MENIIT
NEET | IIT-JEE | FOUNDATION
Corporate Office: 44-A/1, Kalu Sarai, New Delhi 110016 | Web: www.meniit.com

JEE MAINS-2019

11-01-2019 Online (Evening)

IMPORTANT INSTRUCTIONS

1. The test is of 3 hours duration.
2. This Test Paper consists of 90 questions. The maximum marks are 360.
3. There are three parts in the question paper A, B, C consisting of **Mathematics, Chemistry and Physics** having 30 questions in each part of equal weightage. Each question is allotted 4 (four) marks for correct response.
4. Out of the four options given for each question, only one option is the correct answer.
5. For each incorrect response 1 mark i.e. ¼ (one-fourth) marks of the total marks allotted to the question will be deducted from the total score. No deduction from the total score, however, will be made if no response is indicated for an item in the Answer Box.
6. Candidates will be awarded marks as stated above in instruction No.3 for correct response of each question. One mark will be deducted for indicating incorrect response of each question. No deduction from the total score will be made if no response is indicated for an item in the answer box.
7. There is only one correct response for each question. Marked up more than one response in any question will be treated as wrong response and marked up for wrong response will be deducted accordingly as per instruction 6 above.

PART-A-MATHEMATICS

1. $\lim_{x\to 0} \frac{x\cot(4x)}{\sin^2 x \cot^2(2x)}$ is equal to:

(1) 0 (2) 2 (3) 4 (4*) 1

Sol. $\lim_{x\to 0} \frac{x\cot 4x}{\sin^2 x\cot^2 2x} = \lim_{x\to 0}\left(\frac{x}{\tan 4x}\right)\left(\frac{\tan 2x}{\sin x}\right)^2 = \frac{1}{4}\times 4 = 1$

2. All x satisfying the inequality $(\cot^{-1}x)^2 - 7(\cot^{-1}x) + 10 > 0$, lie in the interval:

(1) $(-\infty, \cot 5) \cup (\cot 4, \cot 2)$ (2) $(\cot 2, \infty)$

(3*) $(-\infty, \cot 5) \cup (\cot 2, \infty)$ (4) $(\cot 5, \cot 4)$

Sol. $(\cot^{-1} x)^2 - 7(\cot^{-1} x) + 10 > 0$

$\Rightarrow (\cot^{-1} x - 2)(\cot^{-1} x - 5) > 0$

$\Rightarrow 0 < \cot^{-1} x < 2 \cup \pi > \cot^{-1} x > 5$

$\Rightarrow x \in (-\infty, \cot 5) \cup (\cot 2, \infty)$

3. If a hyperbola has length of its conjugate axis equal to 5 and the distance between its foci is 13, then the eccentricity of the hyperbola is

(1*) $\frac{13}{12}$ (2) 2 (3) $\frac{13}{6}$ (4) $\frac{13}{8}$

Sol. $b = \frac{5}{2}$, 2ae = 13

$b^2 = a^2(e^2 - 1) \Rightarrow \frac{25}{4} = \frac{169}{4} - a^2$

$\Rightarrow a = 6 \Rightarrow e = \frac{13}{12}$

4. If the area of the triangle whose one vertex is at the vertex of the parabola, $y^2 + 4(x - a^2) = 0$ and the other two vertices are the points of intersection of the parabola and y-axis, is 250 s units, then a value of 'a' is:

(1) $5\sqrt{5}$ (2) $5(2^{1/3})$ (3) $(10)^{2/3}$ (4*)
5

Sol. Vertices of the Δ are (0, 2a), (0, – 2a) and $(a^2, 0)$

Area = $\frac{1}{2}\left|4a^3\right| = 250 \Rightarrow 5$

5. Two lines $\frac{x-3}{1}=\frac{y+1}{3}=\frac{z-6}{-1}$ and $\frac{x+5}{7}=\frac{y-2}{-6}=\frac{z-3}{4}$ intersect at the point R. The reflection of R in the xy-plane has coordinates

(1*) (2, –4, –7) (2) (2, 4, 7) (3) (2, –4, 7) (4) (–2, 4, 7)

Sol. $\frac{x-3}{1}=\frac{y+1}{3}=\frac{z-6}{-1}=\lambda$ .... (1)

$\frac{x+5}{7}=\frac{y-2}{-6}=\frac{z-3}{4}=\mu$ .... (2)

Let the point of intersection be

P (λ + 3, 3λ – 1, – λ + 6) $\equiv$ P (7μ – 5, – 6μ + 2, 4μ + 3)

$\left.\begin{matrix}\lambda+3=7\mu-5\\-\lambda+6=4\mu+3\end{matrix}\right]\Rightarrow \mu = 1$

$\Rightarrow$ P (2, – 4, 7) $\Rightarrow$ Required point be (2, – 4, – 7)

6. Contrapositive of the statement

"If two numbers are not equal, then their squares are not equal" is:

(1) If the squares of two numbers are not equal, then the numbers are equal

(2) If the squares of two numbers are equal, then the numbers are not equal

(3*) If the squares of two numbers are equal, then the numbers are equal

(4) If the squares of two numbers are not equal, then the numbers are not equal

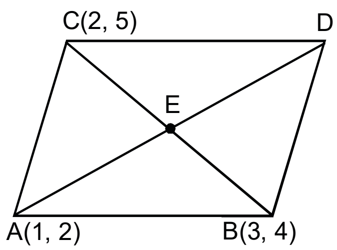
Sol. Contra positive of p $\rightarrow$ q is ~ q $\rightarrow$~ p

$\therefore$ Answer is 3

7. If in a parallelogram ABDC, the coordinates of A, B and C are respectively (1, 2), (3, 4) and (2, 5), then the equation of the diagonal AD is:

(1*) 5x –3y + 1 = 0 (2) 5x +3y – 11 = 0 (3) 3x –5y + 7 = 0 (4) 3x + 5y – 13 = 0

Sol. E is $\left(\frac{5}{2},\frac{9}{2}\right)$



Slope of AD $=\frac{5}{3}$

Equation of AD is $y-2=\frac{5}{3}(x-1)$

$\Rightarrow$ 5x – 3y + 1 = 0

8. The integral $\int_{\pi/6}^{\pi/4}\frac{dx}{\sin 2x\left(\tan^5 x+\cot^5 x\right)}$ equals:

(1) $\frac{1}{20}\tan^{-1}\left(\frac{1}{9\sqrt{3}}\right)$

(2*) $\frac{1}{10}\left(\frac{\pi}{4}-\tan^{-1}\left(\frac{1}{9\sqrt{3}}\right)\right)$

(3) $\frac{\pi}{40}$

(4) $\frac{1}{5}\left(\frac{\pi}{4}-\tan^{-1}\left(\frac{1}{3\sqrt{3}}\right)\right)$

Sol. $I=\int_{\frac{\pi}{6}}^{\frac{\pi}{4}}\frac{dx}{2\sin x\cos x(\tan^5 x+\cot^5 x)}$

$=\int_{\frac{\pi}{6}}^{\frac{\pi}{4}}\frac{\sec^2 x\tan^4 x\,dx}{2(\tan^{10} x+1)}$

Put $\tan^5 x = t$

$\Rightarrow I=\frac{1}{10}\int_{\frac{1}{9\sqrt{3}}}^{1}\frac{dt}{t^2+1}=\frac{1}{10}\left(\tan^{-1}1-\tan^{-1}\frac{1}{9\sqrt{3}}\right)$

9. Let x, y be positive real numbers and m, n positive integers. The maximum value of the expression $\frac{x^m y^n}{\left(1+x^{2m}\right)\left(1+y^{2n}\right)}$ is

(1) 1

(2) $\frac{1}{2}$

(3*) $\frac{1}{4}$

(4) $\frac{m+n}{6mn}$

Sol. $\frac{x^m t^n}{\left(1+x^{2m}\right)\left(1+y^{2n}\right)}$

$=\frac{1}{\left(x^m+\frac{1}{x^m}\right)\left(y^m+\frac{1}{y^n}\right)}$

Put $x^m+\frac{1}{x^m}\geq 2$

$\Rightarrow\frac{1}{\left(x^m+\frac{1}{x^m}\right)}\leq\frac{1}{2}$

$\Rightarrow$ maximum Value $=\frac{1}{4}$

10. Let $S_n = 1 + q + q^2 + \dots\dots\dots\; q^n$ and $T_n = 1 + \left(\frac{q+1}{2}\right)+\left(\frac{q+1}{2}\right)^2+\dots\dots\dots\dots\dots\dots+\left(\frac{q+1}{2}\right)^n$ where q is a real number and $q \neq 1$. If ${}^{101}C_1 + {}^{101}C_2\cdot S_1 + \dots\dots\dots\dots\dots\dots + {}^{101}C_{101}\cdot S_{100} = \alpha\, T_{100}$, then α is equal to

(1) 2^{99} (2) 202 (3) 200 (4*) 2^{100}

Sol. $\sum_{r=1}^{101} {}^{101}C_r S_{r-1}$

$= \sum_{r=1}^{101} {}^{101}C_r \frac{q^r - 1}{q-1}$

$= \frac{1}{q-1}\left(\sum_{r=1}^{101} {}^{101}C_r q^r - \sum_{r=1}^{101} {}^{101}C_r\right)$

$= \frac{1}{q-1}\left((1+q)^{101} - 1 - 2^{101} + 1\right)$

$= \frac{\alpha}{2^{100}}\left(\frac{(1+q)^{101} - 2^{101}}{q-1}\right)$

$\Rightarrow \alpha = 2^{100}$

11. Let α and β be the roots of the quadratic equation $x^2 \sin\theta - x(\sin\theta \cos\theta + 1) + \cos\theta = 0$ $(0 < \theta < 45^\circ)$, and $\alpha < \beta$. Then $\sum_{n=0}^{\infty}\left(\alpha^n + \frac{(-1)^n}{\beta^n}\right)$ is equal to

(1) $\frac{1}{1-\cos\theta} - \frac{1}{1+\sin\theta}$ (2) $\frac{1}{1+\cos\theta} + \frac{1}{1-\sin\theta}$

(3*) $\frac{1}{1-\cos\theta} + \frac{1}{1+\sin\theta}$ (4) $\frac{1}{1+\cos\theta} - \frac{1}{1-\sin\theta}$

Sol. Using quadratic formula,

$$x = \frac{(\cos\theta\sin\theta + 1) \pm \sqrt{(\cos\theta\sin\theta+1)^2 - 4\sin\theta\cos\theta}}{2\sin\theta}$$

$$= \frac{(\cos\theta\sin\theta + 1) \pm (\cos\theta\sin\theta - 1)}{2\sin\theta}$$

$= \cos\theta$, $\text{cosec}\,\theta$

$\alpha = \cos\theta$, $\beta = \text{cosec}\,\theta$

$\therefore \sum_{n=0}^{\infty} \alpha^n + \frac{(-1)^n}{\beta^n}$

$= \sum_{n=0}^{\infty} (\text{cosec})^n + \sum_{n=0}^{\infty} (-\sin\theta)^n$

$= \frac{1}{1-\cos\theta} + \frac{1}{1+\sin\theta}$

$\therefore$ (3) is the correct answer.

12. A bag contains 30 white balls and 10 red balls. 16 balls are drawn one by one randomly from the bag with replacement. If X be the number of white balls drawn, then $\left(\frac{\text{mean of X}}{\text{standard deviation of X}}\right)$ is equal to

(1) 4 (2*) $4\sqrt{3}$ (3) $3\sqrt{2}$ (4) $\frac{4\sqrt{3}}{3}$

Sol. There are 30 white balls and 10 red balls

$$P(\text{white ball}) = \frac{30}{40} = p$$

$$\Rightarrow q = \frac{1}{4}$$

$$\frac{\text{mean}(x)}{\text{standard deviation}(x)} = \frac{np}{\sqrt{npq}}$$

$$= \sqrt{\frac{np}{q}} = \sqrt{\frac{16\times\left(\frac{3}{4}\right)}{\frac{1}{4}}} = 4\sqrt{3}$$

13. Let z be a complex number such that |z| + z = 3 + i (where i = $\sqrt{-1}$). Then |z| is equal to:

(1) $\frac{\sqrt{34}}{3}$ (2*) $\frac{5}{3}$ (3) $\frac{\sqrt{41}}{4}$ (4) $\frac{5}{4}$

Sol. $|z| + z = 3 + i$, Let $z = x + iy$ $(x, y \in R)$

$\Rightarrow y = 1$ and $\sqrt{x^2+1} + x = 3$

$$\Rightarrow x = \frac{4}{3}$$

$$\Rightarrow |z| = \frac{5}{3}$$

14. If $= \begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$ (a + b + c) (x + a + b + c)^2, x $\neq$ 0 and a + b + c $\neq$ 0, then x is equal to

(1) abc (2*) –(a + b + c) (3) 2(a + b + c) (4) –2(a + b + c)

Sol. $D = \begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix} = \begin{vmatrix} -(a+b+c) & 0 & 2a \\ a+b+c & -(a+b+c) & 2b \\ 0 & (a+b+c) & c-a-b \end{vmatrix} =$

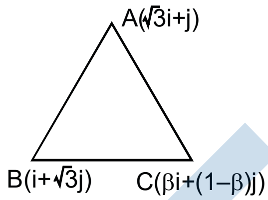
$$= (a+b+c)^2 \begin{vmatrix} -1 & 0 & 2a \\ 1 & -1 & 2b \\ 0 & 1 & c-a-b \end{vmatrix} = (a+b+c)^2 \begin{vmatrix} 0 & 0 & a+b+c \\ 1 & -1 & 2b \\ 0 & 1 & c-a-b \end{vmatrix}$$

$= (a + b + c)^3$

15. Let $\sqrt{3}\,\hat{i} + \hat{j}, \hat{i} + \sqrt{3}\,\hat{j}$, and $\beta\hat{i} + (1-\beta)\hat{j}$ respectively be the position vectors of the points A, B and C w.r.t. the origin O. If the distance of C from the bisector of the acute angle between OA and OB is $\frac{3}{\sqrt{2}}$, then the sum of all positive values of β is:

(1) 4 (2) 3 (3) 2 (4*) 1

Sol. Equation of the angle bisectoer of OA & OB is



$\vec{r} = t(\hat{i} + \hat{j})$

$\Rightarrow \frac{|2\beta - 1|}{\sqrt{2}} = \frac{3}{\sqrt{2}}$

$\Rightarrow \beta = 2, -1$

16. If 19th term of a non-zero A.P. is zero, then its (49th term) : (29th term) is

(1) 4 : 1 (2) 1 : 3 (3*) 3 : 1 (4) 2 : 1

Sol. $a + 18d = 0 \Rightarrow a = -18d$

$\frac{t_{49}}{t_{29}} = \frac{a+48d}{a+28d} = \frac{-18d+48d}{-18d+28d}$

$= \frac{30d}{10d} = 3$

17. If $\int \frac{x+1}{\sqrt{2x-1}}\,dx = f(x)\sqrt{2x-1} + C$, where C is a constant of integration, then f(x) is equal to:

(1) $\frac{1}{3}(x + 1)$ (2) $\frac{2}{3}(x + 2)$ (3) $\frac{2}{3}(x - 4)$ (4*) $\frac{1}{3}(x + 4)$

Sol. I = Put $2x - 1 = t^2 \Rightarrow dx = tdt$

$I = \int\left(\frac{t^2+1}{2} + 1\right)dt = \frac{1}{2}\left(\frac{t^3}{3} + 3t\right)$

$= t\left(\frac{t^2+9}{6}\right) = \sqrt{2x-1}\left(\frac{x+4}{3}\right)$

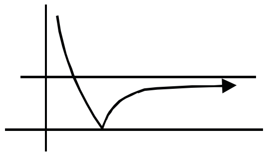
18. Let a function f : $(0, \infty) \to (0, \infty)$ be defined by $f(x) = \left|1 - \frac{1}{x}\right|$. Then *f* is:

(1) not injective but it is surjective (2*) injective only

(3) neither injective nor surjective (4) both injective as well as surjective

Sol. $f(x) = \left|1 - \frac{1}{x}\right| \qquad x > 0$

$$f(x) = \begin{cases} \frac{1}{x} - 1 & x > 0 \\ 1 - \frac{1}{x} & 1 \le x \end{cases}$$



$\Rightarrow$ f (x) is one-one but not onto

19. Let K be the set of all real values of x where the function

f(x) = sin |x| – |x| + 2(x – π) cos |x| is not differentiable. Then the set K is equal to:

(1*) ϕ (empty set) (2) {π} (3) {0} (4) {0, π}

Sol. f (x) = sin | x | – | x | + 2 (x – π) cos x = $\begin{cases} -\sin x + x + 2(x-\pi)\cos x & x < 0 \\ \sin x - x + 2(x-\pi)\cos x & 0 \le x \end{cases}$

at x = 0

LHD = RHD $\Rightarrow$ f (x) is differentiable $\forall$ x $\in$ R

20. The area (in s units) in the first quadrant bounded by the parabola, $y = x^2 + 1$, the tangent to it at the point (2, 5) and the coordinate axes is

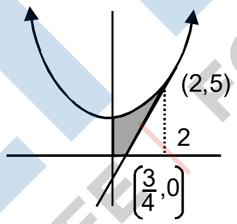
(1) $\frac{8}{3}$ (2*) $\frac{37}{24}$ (3) $\frac{187}{24}$ (4) $\frac{14}{3}$

Sol. Equation of tangent to the parabola $y = x^2 + 1$ at (2, 5) is 4x – y = 3 or y = 4x – 3

Requried area

$$= \int_0^2 (x^2 + 1)dx - \frac{1}{2} \times 5 \times \frac{5}{4}$$

$$= \frac{8}{3} + 2 - \frac{25}{8} = \frac{37}{24}$$



21. Given $\frac{b+c}{11} = \frac{c+a}{12} = \frac{a+b}{13}$ for a Δ ABC with usual notation. If $\frac{\cos A}{\alpha} = \frac{\cos B}{\beta} = \frac{\cos C}{\gamma}$, then the ordered triad (α, β, γ) has a value:

(1*) (7, 19, 25) (2) (3, 4, 5) (3) (5, 12, 13) (4) (19, 7, 25)

Sol. $\frac{b+c}{11} = \frac{c+a}{12} = \frac{a+b}{13} = \frac{a+b+c}{18}$

$\Rightarrow$ a = 7k, b = 6k, c = 5k

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc} = \frac{1}{5}$$

$$\cos B = \frac{19}{25}, \cos C = \frac{5}{7}$$

$$\frac{1}{5\alpha}=\frac{19}{35\beta}=\frac{5}{7\gamma}$$

$$\Rightarrow \frac{7}{35\alpha}=\frac{19}{35\beta}=\frac{25}{35\gamma}$$

$\alpha:\beta:\gamma = 7:19:25$

22. The solution of the differential equation $\frac{dy}{dx}=(x-y)^2$, when y(1) = 1, is

(1) $\log_e\left|\frac{2-x}{2-y}\right|=x-y$

(2*) $-\log_e\left|\frac{1-x+y}{1+x-y}\right|=2(x-1)$

(3) $-\log_e\left|\frac{1+x-y}{1-x+y}\right|=x+y-2$

(4) $\log_e\left|\frac{2-y}{2-x}\right|=2(y-1)$

Sol. $\frac{dy}{dx}=(x-y)^2$

If $x-y=t \Rightarrow \frac{dy}{dx}=1-\frac{dt}{dx}$

$dx=\frac{dt}{1-t^2} \Rightarrow x=\frac{1}{2}\ell n\left|\frac{1+t}{1-t}\right|+c$

$\Rightarrow x=\frac{1}{2}\ell n\left|\frac{1+x-y}{1-x+y}\right|+1$

23. Let the length of the latus rectum of an ellipse with its major axis along x-axis and centre at the origin, be 8. If the distance between the foci of this ellipse is equal to the length of its minor axis, then which one of the following points lies on it ?

(1) $(4\sqrt{2}, 2\sqrt{2})$ (2*) $(4\sqrt{3}, 2\sqrt{2})$ (3) $(4\sqrt{3}, 2\sqrt{3})$ (4) $(4\sqrt{2}, 2\sqrt{3})$

Sol. $\frac{2b^2}{a}=8$ and $ae=b$

$\Rightarrow \frac{a^2(1-e^2)}{a}=4$ and $a^2e^2=a^2(1-e^2)$

$a(1-e^2)=4 \qquad \Rightarrow e=\frac{1}{\sqrt{2}}$

$\Rightarrow a=8$

Ellipse : $\frac{x^2}{64}+\frac{y^2}{32}=1$

24. Let S = (1, 2, ..., 20). A subset B of S is said to be "nice", if the sum of the elements of B is 203. Then the probability that a randomly chosen subset of S is "nice" is:

(1) $\frac{7}{2^{20}}$ (2*) $\frac{5}{2^{20}}$ (3) $\frac{4}{2^{20}}$ (4) $\frac{6}{2^{20}}$

Sol. Total number of subsets = 2^{20} = n (S)

Rejected group of number = (7), (6, 1), (5, 2), (4, 3), (4, 2, 1)

n (E) = 5

P (E) = $\frac{5}{2^{20}}$

25. If the points (2, α, β) lies on the plane which passes through the points (3, 4, 2) and (7, 0, 6) and is perpendicular to the plane 2x – 5y = 15, then $2\alpha - 3\beta$ is equal to:

(1) 12 (2*) 7 (3) 5 (4) 17

Sol. Equation of the plane is

$$\begin{vmatrix} x-3 & y-4 & z-2 \\ 4 & -4 & 4 \\ 2 & -5 & 0 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} x-3 & y-4 & z-2 \\ 1 & -1 & 1 \\ 2 & -5 & 0 \end{vmatrix} = 0 \Rightarrow 5(x-3) + 2(y-4) - 3(z-2) = 0$$

$\Rightarrow 5x + 2y - 3z = 17$

$\Rightarrow 2\alpha - 3\beta = 7$

26. Let $(x + 10)^{50} + (x - 10)^{50} = a_0 + a_1x + a_2x^2 + \ldots\ldots\ldots + a_{50}x^{50}$, for all $x \in R$; then $\frac{a_2}{a_0}$ is equal to

(1) 12.50 (2) 12.00 (3*) 12.25 (4) 12.75

Sol. $(10 + x)^{50} + (10 - x)^{50}$

$a_0 = (10^{50})(2)$

$a_2 = {}^{50}C_2(10)^{48}(2)$

$$\frac{a_2}{a_0} = \frac{{}^{50}C_2(10)^{48}(2)}{10^{52}(2)} = 12.25$$

27. The number of functions *f* from {1, 2, 3, ..., 20} onto {1, 2, 3, ..., 20} such that *f* (k) is a multiple of 3, whenever k is a multiple of 4, is

(1) $6^5 \times (15)!$ (2) $5! \times 6!$ (3*) $(15)! \times 6!$ (4) $5^6 \times 15$

Sol. f : {1, 2, 3 ...., 20} onto {1, 2, 3 ...., 20} such that f (4k) 3t, where k, t $\in$ N

Number of function = ${}^6P_5 \cdot 15! = 6!\ 15!$

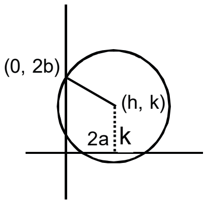
28. A circle cuts a chord of length 4a on the x-axis and passes through a point on the y-axis, distant 2b from the origin. Then the locus of the centre of this circle, is

(1) a hyperbola (2) an ellipse (3) a straight line (4*) a parabola

Sol. $h^2 + (k - 2b)^2 = k^2 + 4a^2$

$\Rightarrow x^2 = 4by + 4a^2 - 4b^2$

locus is a parabola



29. Let $f(x) = \frac{x}{\sqrt{a^2+x^2}} - \frac{d-x}{\sqrt{b^2+(d-x)^2}}$, $x \in R$ where a, b and d are non-zero real constants. Then:

(1*) f is an increasing function of x

(2) f is a decreasing function of x

(3) f ′ is not a continuous function of x

(4) f is neither increasing nor decreasing function of x

Sol. $f(x) = \frac{x}{\sqrt{a^2+x^2}} - \frac{d-x}{\sqrt{b^2+(d-x)^2}}$

$f(x) = \frac{x}{\sqrt{a^2+x^2}} + \frac{x-d}{\sqrt{b^2+(x-d)^2}}$

$= \sin\tan^{-1}\frac{x}{a} + \sin\tan^{-1}\frac{x-d}{b}$

$\Rightarrow$ f (x) is an increasing function

30. Let A and B be two invertible matrices of order 3 × 3. If det $(ABA^T) = 8$ and det $(AB^{-1}) = 8$, then det $(BA^{-1}B^T)$ is equal to :

(1) $\frac{1}{4}$ (2) 1 (3*) $\frac{1}{16}$ (4) 16

Sol. $|ABA^T| = 8 \Rightarrow |A|^2 |B| = 8$

$|AB^{-1}| = 8 \Rightarrow |A| = 8|B|$

$\Rightarrow |B|^3 = \frac{1}{8} \Rightarrow |B| \frac{1}{2} = \& |A| = 4$

$|BA^{-1}B^T| = \frac{|B|^2}{|A|} = \frac{1}{16}$

PART-B-CHEMISTRY

31. The reaction,

$MgO (s) + C(s) \rightarrow Mg(s) + CO(g)$, for which $\Delta_r H° = +491.1$ kJ mol^{-1} and $\Delta_r S° = 198.0$ JK^{-1} mol^{-1}, is not feasible at 298 K. Temperature above which reaction will be feasible is

(1) 2040.5 K (2) 1890.0 K (3) 2480.3 K (4*) 2380.5 K

Sol. In order to be spontaneous $\Delta G°$ should be –ve

$\Delta G° = \Delta H° - T\Delta S°$

$0 = 491.1 \times 10^3 - T \times 198$

$$T = \frac{491100}{198} = 2480$$

If temp is above 2480 K, the reaction will be spontaneous.

32 The correct match between item I and item II is:

Item–I	**Item–II**
(A) Allosteric effect	(P) Molecule binding to the active site of enzyme
(B) Competitive inhibitor	(Q) Molecule crucial for communication in the body.
(C) Receptor	(R) Molecule binding to a site other than the active site of enzyme.
(D) Poison	(S) Molecule binding to the enzyme covalently

(1*) (A)→(R); (B)→(P); (C)→(Q); (D)→(S) (2) (A)→(P); (B)→(R); (C)→(Q); (D)→(S)

(3) (A)→(R); (B)→(P); (C)→(S); (D)→(Q) (4) (A)→(P); (B)→(R); (C)→(S); (D)→(Q)

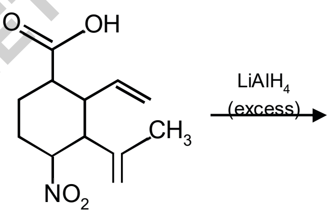
Sol. Fact based, go through definition.

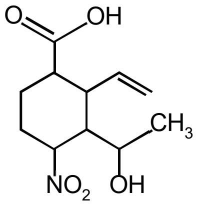
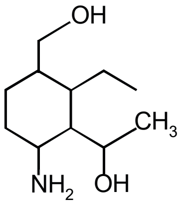
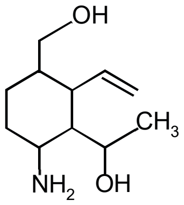
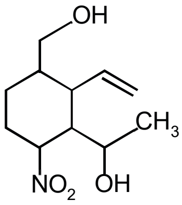
33 The coordination number of Th in $K_4[Th(C_2O_4)_4(OH_2)_2]$ is: ($C_2O_4^{2-}$ = Oxalato)

(1) 14 (2) 6 (3) 8 (4*) 10

Sol. Th is a metal having large size and oxalate is a bidentate ligand hence its co-ordination number in given complex is 10.

34 The major product obtained in the following reaction is



(1)  (2)  (3)  (4*) 

Sol. Since $LiAlH_4$ is a strong reducing agent, it will reduce $-NO_2$, $-\overset{O}{\overset{||}{C}}-O-H$ and ketonic group but cannot reduce normal alkene >C=C<

35. The standard reaction Gibbs energy for a chemical reaction at an absolute temperature T is given by

$\Delta_r G° = A - BT$

Where A and B are non-zero constants. Which of the following is **TRUE** about this reaction?

(1*) Endothermic if A > 0
(2) Exothermic if A > 0 and B < 0
(3) Endothermic if A < 0 and B > 0
(4) Exothermic if B < 0

Sol. $\Delta G^{\circ} = \Delta H^{\circ} - T\Delta S^{\circ}$

$\Delta G^{\circ} = A - BT$

In endothermic reaction ΔH = +ve. Hence, A = +ve

36. The radius of the largest sphere which fits properly at the centre of the edge of a body centered cubic unit cell is : (Edge length is represented by 'a')

(1) 0.027 a
(2) 0.047 a
(3) 0.134 a
(4*) 0.067 a

Sol. $\sqrt{3}a = 4R$

$$R = \frac{\sqrt{3}a}{4}$$

$2(R + r) = a$

$$2\left(\frac{\sqrt{3}a}{4} + r\right) = a$$

$$\frac{\sqrt{3}a}{2} + 2r = a$$

$$2r = a - \frac{\sqrt{3}a}{2} = \frac{2a - \sqrt{3}a}{2}$$

$$r = \frac{2a - 1.7329}{4} = \frac{.268a}{4} = .067a$$

37 The hydride that is NOT electron deficient is:

(1*) SiH_4 (2) B_2H_6 (3) GaH_3 (4) AlH_3

Sol. SiH_4 has complete octet hence it is not an electron deficient hydride.

38. Given the equilibrium constant:

K_C of the reaction:

$Cu(s) + 2Ag^+ (aq) \rightarrow Cu^{2+} (aq) + 2Ag (s)$ is 10×10^{15}, calculate the E^0_{cell} of this reaction at 298 K

$[2.303 \frac{RT}{F}$ at 298 K = 0.059 V]

(1) 0.04736 mV (2) 0.4736 mV (3*) 0.4736 V (4) 0.04736 V

Sol. $\Delta G^\circ = -nFE^0_{cell} = -2.303RT \log K_{eq}$

$2FE^o_{cell} = 2.303RT \log K_{eq}$

$$2E^o_{cell} = \frac{2.303RT}{F} \log 10 \times 10^{15}$$

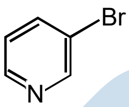
$$E^o_{cell} = \frac{.059}{2} \times 16 = 8 \times .059 \Rightarrow .472$$

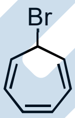
39. The correct option with respect to the Pauling electronegativity values of the elements is:

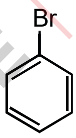
(1) Te > Se (2) Ga < Ge (3) Si < Al (4) P > S

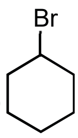
Sol. Electronegativity increases from left to right in a period and decreases down the group.

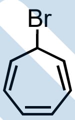
40. Which of the following compounds will produce a precipitate with $AgNO_3$?

(1) 

(2*) 

(3) 

(4) 

Sol. On ionization 

, this compound will produce Aromatic cation, which is stable.

41. The de Broglie wavelength (λ) associated with a photoelectron varies with the frequency (v) of the incident radiation as, [v_0 is threshold frequency]

(1) $\lambda \propto \frac{1}{(v - v_0)}$ (2) $\lambda \propto \frac{1}{(v - v_0)^{\frac{1}{4}}}$ (3) $\lambda \propto \frac{1}{(v - v_0)^{\frac{3}{2}}}$ (4*) $\lambda \propto \frac{1}{(v - v_0)^{\frac{1}{2}}}$

Sol. $\lambda = \frac{h}{mv}$

According to Einstein's theory of photoelectric effect:

$h\nu = h\nu_0 + KE$

$$h\nu = h\nu_o + \frac{1}{2}mv^2$$

$$2h(\nu - \nu_o) = mv^2$$

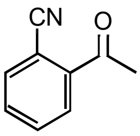
$$\frac{2h(\nu - \nu_0)}{m} = v^2$$

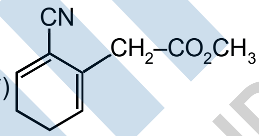
$$v \propto (\nu - \nu_o)^{\frac{1}{2}}$$

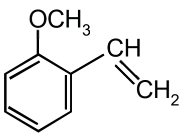
$$\lambda \propto \frac{h}{m(\nu - \nu_o)^{\frac{1}{2}}}$$

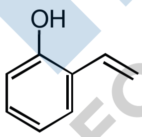
$$\lambda \propto \frac{1}{(\nu - \nu_o)^{\frac{1}{2}}}$$

42. Which of the following compounds reacts with ethyl magnesium bromide and also decolourises bromine water solution:

(1) 

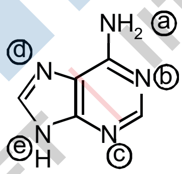
(2*) 

(3*) 

(4) 

Sol. Option B and option D both will react with Grignard reagent and decolorizes Br_2/H_2O.(IIT has given option D only)

43. In the following compound



the favourable site (s) for protonation is(are)

(1) (a) & (e) (2*) (b), (c) & (d) (3) (a) & (d) (4) (a)

Sol. After protonation at b or c or d the conjugate acid is stabilized by resonance.

44. Taj Mahal is being slowly disfigured and discoloured. This is primarily due to:

(1) global warming (2*) acid rain (3) water pollution (4) soil pollution

Sol. Acid rain reacts with marble. Hence, The Taj Mahal which made up of marble is discolored.

45. The relative stability of +1 oxidation state of group 13 elements follows the order:

(1) Al < Ga < Tl < In (2) Tl < In < Ga < Al (3) Ga < Al < In < Tl (4*) Al < Ga < In < Tl

Sol. Inert pair effect gradually increases down the group. Hence, stability of lower oxidation state increases down the group.

46. For the equilibrium,

$2\,H_2O \rightleftharpoons H_3O^+ + OH^-$, the value of ΔG^o at 298 K is approximately

(1) 100 kJ mol^{-1} (2) –80 kJ mol^{-1} (3*) 80 kJ mol^{-1} (4) –100 kJ mol^{-1}

Sol. $\Delta G° = -2.303RT\log K_{eq}$

$= -2.303 \times 8.314 \times 298 \log 10^{-14}$

$= -2.303 \times 8.314 \times 298 \times -14$

= 79,881.87

$\approx$ 80 KJ mol^{-1}

47. The reaction that does NOT define calcination is:

(1) $Fe_2O_3 \cdot X\,H_2O \xrightarrow{\Delta} Fe_2O_3 + X\,H_2O$ (2*) $2Cu_2S + 3\,O_2 \xrightarrow{\Delta} 2Cu_2O + 2\,SO_2$

(3) $ZnCO_3 \xrightarrow{\Delta} ZnO + CO_2$ (4) $CaCO_3 \cdot MgCO_3 \xrightarrow{\Delta} CaO + MgO + 2CO_2$

Sol. Calcination takes place in absence of air. Hence step 2 is not defining it.

48. A compound 'X' on treatment with Br_2/NaOH, provided C_3H_9N, which gives positive carbylamine test. Compound 'X' is:

(1) $CH_3COCH_2NHCH_3$ (2) $CH_3CH_2COCH_2NH_2$

(3*) $CH_3CH_2CH_2CONH_2$ (4) $CH_3CON(CH_3)_2$

Sol.. Br_2/NaOH converts amide into primary amine having one carbon atom less, which gives carbylamines test.

49. Among the colloids cheese (C), milk (M) and smoke (S), the correct combination of the dispersed phase and dispersion medium, respectively is:

(1) C: liquid in solid; M : liquid in solid; S: solid in gas

(2*) C: liquid in solid; M : liquid in liquid; S: solid in gas

(3) C: solid in liquid; M : liquid in liquid; S: gas in solid

(4) C: solid in liquid; M : solid in liquid; S: solid in gas

Sol. Go through different types of colloid and their examples.

50 The homopolymer formed from 4–hydroxy–butanoic acid is

(1) $\left[-O\overset{O}{\overset{\|}{C}}-(CH_2)_3-O-\right]_n$ (2) $\left[-\overset{O}{\overset{\|}{C}}(CH_2)_2-\overset{O}{\overset{\|}{C}}-\right]_n$ (3) $\left[-\overset{O}{\overset{\|}{C}}(CH_2)_2\overset{O}{\overset{\|}{C}}-O-\right]_n$ (4*) $\left[-\overset{O}{\overset{\|}{C}}(CH_2)_3-O-\right]_n$

Sol. 4-hydroxy butanoic acid undergoes intermolecular esterification to give polymer.

51. K_2HgI_4 is 40% is ionised in aqueous solution. The value of its van't Hoff factor (i) is

(1) 1.6 (2*) 1.8 (3) 2.0 (4) 2.2

Sol. $\underset{1-\alpha}{K_2[HgI_4]} \rightleftharpoons \underset{2\alpha}{2K^+} + \underset{\alpha}{[HgI_4]^{2-}}$

Total number of particle = $1 + 2\alpha$

Hence, Van't Hoff factor $= \dfrac{1+2\alpha}{1}$

$= \dfrac{1+2\times 0.4}{1} = 1+0.8 \Rightarrow 1.8$

52. 25 ml of the given HCl solution requires 30 mL of 0.1 M sodium carbonate solution. What is the volume of this HCl solution requried to titrate 30 mL of 0.2 M aqueous NaOH solution?

(1*) 25 mL (2) 75 mL (3) 50 mL (4) 12.5 mL

Sol. Apply law of equivalence:

$25 \times N = 30 \times 0.1 \times 2$

$N_{HCl} = \dfrac{30\times 0.2}{25} = \dfrac{6}{5}\times 0.2 = \dfrac{1.2}{5}$

For the 2nd titration

$\dfrac{1.2}{5}\times V_{HCl} = 30\times 0.2$

$V_{HCl} = \dfrac{6\times 5}{1.2} = \dfrac{30}{1.2} = 25$ ml

53. The reaction $2X \rightarrow B$ is a zeroth order reaction. If the initial concentration of X is 0.2 M, the half life is 6h. When the initial concentration of X is 0.5 M, the time required to reach its final concentration of 0.2 M will be

(1) 9.0 h (2) 12.0 h (3*) 18.0 h (4) 7.2 h

Sol. For zero order reaction

$C_o - C_t = Kt$

$0.5M - 0.2M = Kt$

$0.3 = Kt$ ----- (1)

'K' can be calculated by

$t_{1/2} = \dfrac{C_o}{2K}$

$$6 = \frac{0.2}{2K}$$

$$K = \frac{0.2}{12} = \frac{2\times10^{-1}}{12} = \frac{1}{60}$$

Putting the value of K in eq (1)

$$t = \frac{0.3}{K} = \frac{0.3}{\frac{1}{60}} = 60\times0.3 = 18 \text{ Hr}$$

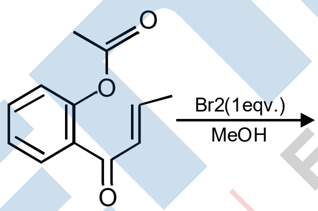
54. Match the following items in Column I with the corresponding items in Column II

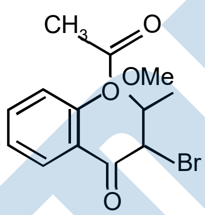
Column–I	Column–II
(i) $Na_2CO_3 \cdot 10H_2O$	(A) Portland cement ingredient
(ii) $Mg(HCO_3)_2$	(B) Castner–Kellner process
(iii) NaOH	(C) Solvay process
(iv) $Ca_3Al_2O_6$	(D) Temporary hardness

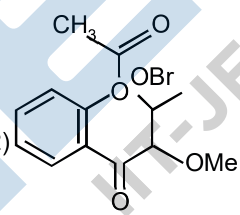
(1) (i)→(B); (ii)→(C); (iii)→(A); (iv)→(D)
(2) (i)→(C); (ii)→(B); (iii)→(D); (iv)→(A)
(3) (i)→(D); (ii)→(A); (iii)→(B); (iv)→(C)
(4*) (i)→(C); (ii)→(D); (iii)→(B); (iv)→(A)

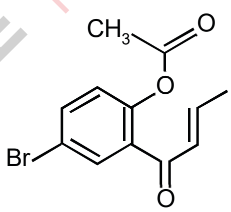
Sol. Fact based

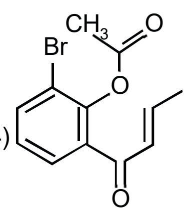
55. The major product obtained in the following conversion is



(1*) 

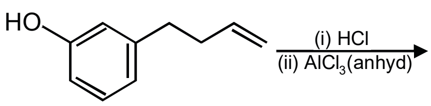
(2) 

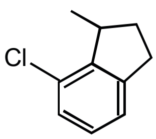
(3) 

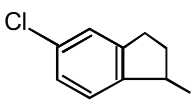
(4) 

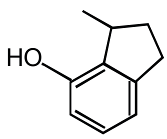
Sol. Attack on $>C=C<$ is more preferred than benzene ring.

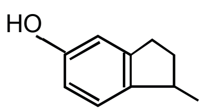
56. The major product of the following reaction is:



(1) 

(2) 

(3) 

(4*) 

Sol. First Markonikov's addition one alkene followed by intramolecular Friedel-Craft alkylation takes place.

57. The higher concentration of which gas in air can cause stiffness of flower buds?

(1) NO_2 (2) CO_2 (3*) SO_2 (4) CO

Sol. SO_2 gas causes stiffness of flower bunds?

58. The correct match between items I and II is:

Item–I	**Item–II**
(A) Ester Test	(P) Tyr
(B) Carbylamine test	(Q) Asp
(C) Phthalein dye test	(R) Ser
	(S) Lys

(1*) (A)→(Q); (B)→(S); (C)→(P) (2) (A)→(R); (B)→(Q); (C)→(P)

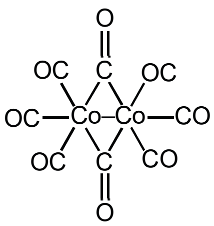
(3) (A)→(R); (B)→(S); (C)→(Q) (4) (A)→(Q); (B)→(S); (C)→(R)

Sol. Go through structure of amino acids.

59. The number of bridging CO ligand(s) and Co–Co bond(s) in $Co_2(CO)_8$, respectively.

(1*) 2 & 1 (2) 2 & 0 (3) 0 & 2 (4) 4 & 0

Sol. Go through structure of $[Co_2(CO)_8]$



60. $\underline{A} \xrightarrow{4KOH,\ O_2} 2\underset{(green)}{\underline{B}} + 2H_2O$

$3\underline{B} \xrightarrow{4HCl} 2\underset{(purple)}{\underline{C}} + MnO_2 + 2H_2O$

$2\underline{C} \xrightarrow{H_2O,\ KI} 2\underline{A} + 2KOH + \underline{D}$

In the above sequence of reactions, $\underline{A}$ and $\underline{D}$,respectively are

(1) KI and $KMnO_4$ (2*) MnO_2 and KIO_3 (3) KIO_3 and MnO_2 (4) KI and K_2MnO_4

Sol. $MnO_2 \xrightarrow{4KOH,\ O_2} \underset{(Green)}{2K_2MnO_4} + 2H_2O$

$3K_2MnO_4 \xrightarrow{4HCl} \underset{(Purple)}{2KMnO_4} + MnO_2 + 2H_2O$

$3KMnO_4 \xrightarrow{H_2O,\ KI} 2MnO_2 + 2KOH + KIO_3$

PART-C-PHYSICS

61. A particle moves from the point $\left(2.0\hat{i}+4.0\hat{j}\right)$m, at t = 0, with an initial velocity $\left(5.0\hat{i}+4.0\hat{j}\right)$ ms^{-1}. It is acted upon by a constant force which produces a constant acceleration $\left(4.0\hat{i}+4.0\hat{j}\right)$ ms^{-2}. What is the distance of the particle from the origin at time 2s?

(1) 15 m (2*) $20\sqrt{2}$m (3) 5 m (4) $10\sqrt{2}$ m

Sol. $\vec{S}=\left(5\hat{i}+4\hat{j}\right)2+\frac{1}{2}\left(4\hat{j}+4\hat{j}\right)4$

$=10\hat{i}+8\hat{j}+8\hat{i}+8\hat{j}$

$\vec{r}_2-\vec{r}_1=18\hat{i}+16\hat{j}$

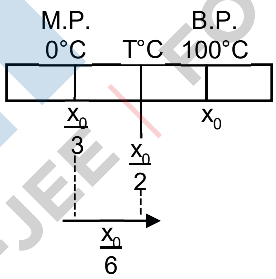
$\vec{r}_2=20\hat{i}+20\hat{j}$

$\left|\vec{r}_f\right|=20\sqrt{2}$

62. A thermometer graduated according to a linear scale reads a value x_0 , when in contact with boiling water and x_0 /3 when in contact with ice. What is the temperature of an object in ºC, if this thermometer in the contact with the object reads $x_0/2$?

(1*) 25 (2) 60 (3) 40 (4)
35

Sol. $t=\frac{x_t-x_0}{x_{100}-x_0}100°C$



$\frac{\frac{x_0}{2}-\frac{x_0}{3}}{x-\frac{x_0}{3}}100°C$

= 25°C

63. A galvanometer having a resistance of 20Ω and 30 divisions on both sides has figure of merit 0.005 ampere/division. The resistance that should be connected in series such that it can be used as a voltmeter upto 15 volt, is

(1) 100 Ω (2) 120 Ω (3*) 80 Ω (4) 125 Ω

Sol. R_g = 20 Ω

$N_L = N_g = N = 30$

$FOM=\frac{1}{\phi}=0.005\ A/Div.$

Current sentivity $= CS = \left(\frac{1}{0.005}\right) = \frac{\phi}{1}$

$I_{gmax} = 0.005 \times 30$

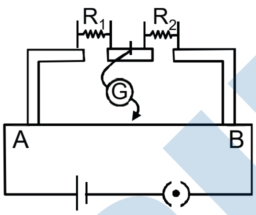
$= 15 \times 10^{-2} = 0.15$

$15 = 0.15[20 + R]$

$100 = 20 + R$

$R = 80.$

64. In the experimental set up of metre bridge shown in the figure, the null point is obtained at a distance of 40 cm from A. If a 10Ω resistor is connected in series with R_1, the null point shifts by 10 cm. The resistance that should be connected in parallel with $(R_1 + 10)\Omega$ such that the null point shifts back to its initial position is



(1) 20 Ω (2) 40 Ω (3*) 60 Ω (4) 30 Ω

Sol. $\frac{R_1}{R_2} = \frac{2}{3}$

$\frac{R_1 + 10}{R_2} = 1$

$\Rightarrow \quad R_1 + 10 = R_2 \quad$ ....(i)

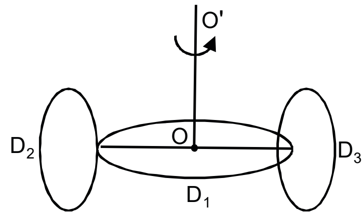
$\frac{2R_2}{3} + 10 = R_2 \quad ; \quad 10 = \frac{R_2}{3}$

$\Rightarrow \quad R_2 = 30\Omega$ & $R_1 = 20\Omega$

$$\frac{\frac{30 \times R}{30 + R}}{30} = \frac{2}{3}$$

$R = 60\ \Omega$

65. A circular disc D_1 of mass M and radius R has two identical discs D_2 and D_3 of the same mass M and radius R attached rigidly at its opposite ends (as shown in figure). The moment of inertia of the system about the axis OO′, passing through the centre of D_1, as shown in the figure, will be



(1) MR^2 (2*) $3MR^2$ (3) $\frac{4}{5}MR^2$ (4) $\frac{2}{3}MR^2$

Sol. $I = \frac{MR^2}{2} + 2\left(\frac{MR^2}{4} + MR^2\right)$

$= \frac{MR^2}{2} + \frac{MR^2}{2} 2MR^2 = 3MR^2$

66. The magnitude of torque on a particle of mass 1 kg is 2.5 Nm about the origin. If the force acting on it is 1 N, and the distance of the particle from the origin is 5m, the angle between the force and the position vector is (in radians):

(1*) $\pi/6$ (2) $\pi/3$ (3) $\pi/8$ (4) $\pi/4$

Sol. $2.5 = 1 \times 5 \sin\theta$

$\sin\theta = 0.5 = \frac{1}{2}$

$\theta = \frac{\pi}{6}$

67. A copper wire is wound on a wooden frame, whose shape is that of an equilateral triangle. If the linear dimension of each side of teh frame is increased by a factor of 3, keeping the number of turns of the coil per unit length of the frame the same, then the self inductance of the coil:

(1) decreases by a factor of 9 (2) increases by a factor of 27

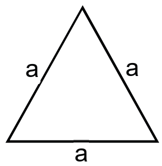
(3*) increases by a factor of 3 (4) decreases by a factor of $9\sqrt{3}$

Sol. Total length L will remain constant

= (3a) N (N = total turns)

And length of winding = (d) N = ℓ

(d = diameter of wire)



Self inductance = $\mu_0 n^2 A\ell$

$= \mu_0 n^2 \left(\frac{\sqrt{3}a^2}{4}\right) dN$

$\propto$ a2 N $\propto$ a

So self inductance will become 3 times.

68. A particle of mass m is moving in a straight line with momentum p. Starting at time t = 0, a force F = kt acts in the same direction on the moving particle during time interval T so that its momentum charges from p to 3p. Here k is a constant. The value of T is

(1) $2\sqrt{\frac{k}{p}}$ (2*) $2\sqrt{\frac{p}{k}}$ (3) $\sqrt{\frac{2k}{p}}$ (4) $\sqrt{\frac{2p}{k}}$

Sol. $\frac{dp}{dt} = F = kt$

$\int_P^{3P} dp = \int_0^T kt\, dp$

$2p = \frac{KT^2}{2}$; $T = 2\sqrt{\frac{p}{k}}$

69. A paramagnetic substance in the form of a cube with sides 1 cm has a magnetic dipole moment of 20 × 10^{-6} J/T when a magnetic intensity of 60×10^3 A/m is applied. Its magnetic susceptibility is:

(1) 3.3×10^{-2} (2) 4.3×10^{-2} (3) 2.3×10^{-2} (4*) 3.3×10^{-4}

Sol. $\chi = \frac{I}{H}$

$I = \frac{\text{Magnetic moment}}{\text{Volume}}$

$I = \frac{20\times10^{-6}}{10^{-6}} = 20\ N/m^2$

$\chi = \frac{20}{60\times10^3} = \frac{1}{3}\times10^{-3}$

$= 0.33 \times 10^{-3} = 3.3 \times 10^{-4}$

70. A simple pendulum of length 1 m is oscillating with an angular frequency 10rad/s. The support of the pendulum starts oscillating up and down with a small angular frequency of 1 rad/s and an amplitude of 10^{-2}m. The relative change in the angular frequency of the pendulum is best given by

(1*) 10^{-3} rad/s (2) 1 rad/s (3) 10^{-1} rad/s (4) 10^{-5} rad/s

Sol. Angular frequency of pendulum

$\omega \propto \sqrt{\frac{g_{eff}}{\ell}}$

$\therefore \quad \frac{\Delta\omega}{\omega} = \frac{1}{2}\frac{\Delta g_{eff}}{g_{eff}}$

$$\Delta\omega = \frac{1}{2}\frac{\Delta g}{g} \times \omega$$

$$= \frac{1}{2} \times \frac{2(A\omega_5^s)}{10}$$

$$\Rightarrow \frac{\Delta\omega}{\omega} = \frac{1 \times 10^{-2}}{10} = 10^{-3}$$

71. The circuit shown below contains two ideal diodes, each with a forward resistance of 50Ω. If the battery voltage is 6V, the current through the 100Ω reistance (in Amperes) is:

(1) 0.036 (2*) 0.020 (3) 0.027 (4) 0.030

Sol. $I = \frac{6}{300} = 0.002$ (D_2 is in reverse bias)

72. An electric field of 1000 V/m is applied to an electric dipole at angle of 45º. The value of electric dipole moment is 10^{-29} cm. What is the potential energy of the electric dipole?

(1) -20×10^{-18} J (2*) -7×10^{-27} J (3) -10×10^{-29} J (4) -9×10^{-20} J

Sol. $U = -\vec{P}.\vec{E}$

$= -PE \cos\theta$

$= -(10^{-29})(10^{3}) \cos 45°$

$= -0.707 \times 10^{-26}$ J

$= -7 \times 10^{-27}$ J

73. A metal ball of mass 0.1 kg is heated upto 500ºC and dropped into a vessel of heat capacity 800 JK^{-1} and containing 0.5 kg water. The initial temperature of water and vessel is 30ºC. What is the approximate percentage increment in the temperature of the water? [Specific heat capacities of water and metal are, respectively, 4200 JK^{-1} and 400 J $kg^{-1}K^{-1}$]

(1) 15% (2) 30% (3) 25% (4*) 20%

Sol. $0.1 \times 400 \times (500 - T) = 0.5 \times 4200 \times (T - 30) + 800 (T - 30)$

$\Rightarrow 40(500 - T) = (T - 30)(2100 + 800)$

$\Rightarrow 20000 - 40T = 2900\,T - 30 \times 2900$

$\Rightarrow 20000 + 30 \times 2900 = T(2940)$

T = 30.4°C]

$$\frac{\Delta T}{T} \times 100 = \frac{6.4}{30} \times 100 = 20\%$$

74. The region between y = 0 and y = d contains a magnetic field $\vec{B} = B\hat{z}$. A particle of mass m and charge q enters the region with a velocity. $\vec{v} = v\hat{i}$ .If $d = \frac{mv}{2qB}$, the acceleration of the charged particle at the point of its emergence at the other side is:

(1) $\frac{qvB}{m}\left(\frac{1}{2}\hat{i} - \frac{\sqrt{3}}{2}\hat{j}\right)$ (2) $\frac{qvB}{m}\left(\frac{\sqrt{3}}{2}\hat{i} + \frac{1}{2}\hat{j}\right)$ (3) $\frac{qvB}{m}\left(\frac{-\hat{j}+\hat{i}}{\sqrt{2}}\right)$ (4) $\frac{qvB}{m}\left(\frac{\hat{i}+\hat{j}}{\sqrt{2}}\right)$

Ans. Bonus

75. A pendulum is executing simple harmonic motiona and its maximum kinetic energy is K_1. If the length of the pendulum is doubled and it performs simple harmonic motion with the same amplitude as in the first case, its maximum kinetic energy is K_2. Then:

(1*) $K_2 = 2K_1$ (2) $K_2 = \frac{K_1}{2}$ (3) $K_2 = \frac{K_1}{4}$ (4) $K_2 = K_1$

Sol. Maximum kinetic energy at lowest point B is given by

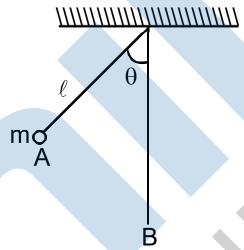
$K = mg\ell\,(1 - \cos\theta)$

where θ = angular amp.

$K_1 = mg\,\ell\,(1 - \cos\theta)$

$K_2 = mg(2\ell)\,(1 - \cos\theta)$

$K_2 = 2K_1$



76. Two rods A and B of identical dimensions are at temperature 30ºC. If A is heated upto 180ºC and B upto TºC, then the new lengths are the same. If the ratio of the coefficients of linear expansion of A and B is 4 : 3, then the value of T is

(1*) 230ºC (2) 270ºC (3) 200ºC (4) 250ºC

Sol. $\Delta\ell_1 = \Delta\ell_2$

$\Delta\alpha_1\Delta T_1 = \ell\alpha_2\Delta T_2$

$\frac{\alpha_1}{\alpha_2} = \frac{\Delta T_1}{\Delta T_2}$; $\frac{4}{3} = \frac{T-30}{180-30}$

T = 230°C

77. If speed (V), acceleration (A) and force (F) are considered as fundamental units, the dimension of Young's modulus will be

(1) $V^{-2}A^2F^{-2}$ (2) $V^{-2}A^2F^2$ (3) $V^{-2}A^{-2}F$ (4*) $V^{-4}A^2F$

Sol. $\frac{F}{A} = y.\frac{\Delta\ell}{\ell}$; $[Y] = \frac{F}{A}$

Now from dimension

$$\frac{F}{A} = \frac{ML}{T^2} \quad ; \quad L = \frac{F}{M} \cdot T^2$$

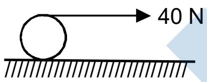
$$L^2 = \frac{F^2}{M^2}\left(\frac{V}{A}\right)^4 \quad \because T = \frac{V}{A}$$

$$L^2 = \frac{F^2}{M^2A^2}\frac{v^2}{A^2} \quad F = MA$$

$$L^2 = \frac{V^4}{A^2}$$

$$[Y] = \frac{[F]}{[A]} = F^1V^{-4}A^2$$

78. A string is wound arond a hollow cylinder of mass 5 kg and radius 0.5 m. If the string is now pulled with a horizontal force of 40 N, and the cylinder is rolling without slipping on a horizontal surface (as shown in the figure), then the angular acceleration of the cylinder will be (Neglect the mass and thickness of the string)

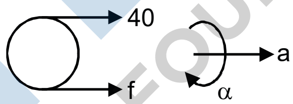


(1) 20 rad/s^2 (2*) 16 rad/s^2 (3) 12 rad/s^2 (4) 10 rad/s^2

Sol. $40 + f = m(R\alpha)$ ...(i)

$40 \times R - f \times R = mR=2^{\alpha}$

$40 - f = mR\alpha$ ...(ii)



From (i) and (ii)

$$\alpha = \frac{40}{mR} = 16$$

79. A 27 mW laser beam has a cross-sectional area of 10mm^2. The magnitude of the maximum electric field in this electromagnetic wave is given by:

[Given permittivity of space ε_0 = 9 × 10^{-12} SI units, Speed of light c = 3 × 10^8 m/s]

(1) 2 kV/m (2) 0.7 kV/m (3) 1 kV/m (4*) 1.4 kV/m

Sol. Intensity of EM wave is given by

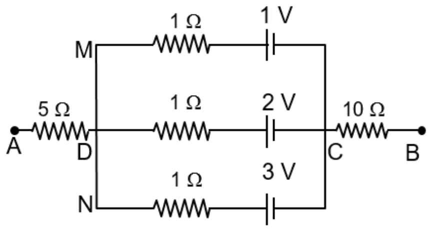
$$I = \frac{\text{Power}}{\text{Area}} = \frac{1}{2}\varepsilon_0 E_0^2 C$$

$$= \frac{27\times10^{-3}}{10\times10^{-6}} = \frac{1}{2}\times9\times10^{-2}\times E^2\times3\times10^8$$

$$E = \sqrt{2}\times10^3 \text{kV / m}$$

= 1.4 kV/m

80. In the circuit shown, the potential difference between A and B is:



(1) 1 V (2*) 2 V (3) 3 V (4) 6 V

Sol. Potential difference across AB will be equal to battery equivalent across CD.

$$V_{AB} = V_{CD} = \frac{\frac{E_1}{r_1}+\frac{E_2}{r_2}+\frac{E_3}{r_3}}{\frac{1}{r_1}+\frac{1}{r_2}+\frac{1}{r_3}} = \frac{\frac{1}{1}+\frac{2}{1}+\frac{3}{1}}{\frac{1}{1}+\frac{1}{1}+\frac{1}{1}}$$

$$= \frac{6}{3} = 2V$$

81. The mass and the diameter of a planet are three times the respective values for the Earth. The period of oscillation of a simple pendulum on the Earth is 2s. The period of oscillation of the same pendulum on the planet would be:

(1) $\frac{\sqrt{3}}{2}$s (2) $\frac{2}{\sqrt{3}}$s (3) $\frac{3}{2}$s (4*) $2\sqrt{3}$s

Sol. $\because \quad g = \frac{GM}{R^2}$

$$\frac{g_p}{g_e} = \frac{M_p}{M_e}\left(\frac{R_e}{R_p}\right)^2 = 3\left(\frac{1}{3}\right)^2 = \frac{1}{3}$$

Also, $T \propto \frac{1}{\sqrt{g}}$

$$\Rightarrow \frac{T_p}{T_e} = \sqrt{\frac{g_e}{g_p}} = \sqrt{3}$$

$$\Rightarrow T_p = 2\sqrt{3}s$$

82. An amplitude modulated signal is plotted below:

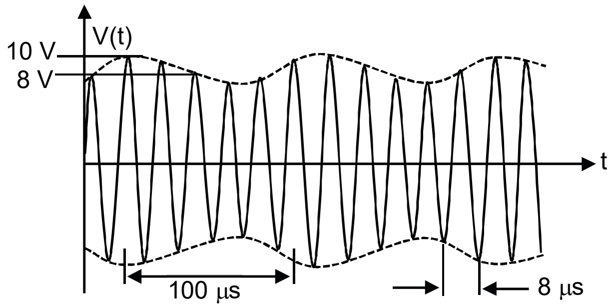
Which one of the following best describes the above signal?

(1) $(9 + \sin(2.5\pi \times 10^5 t)) \sin (2\pi \times 10^4 t)$ V

(2) $(1 + 9 \sin(2\pi \times 10^4 t)) \sin (2.5 \pi \times 10^5 t)$ V

(3*) $(9 + \sin(2\pi \times 10^4 t)) \sin (2.5\pi \times 10^5 t)$ V

(4) $(9 + \sin(4\pi \times 10^4 t)) \sin (5\pi \times 10^5 t)$ V



Sol. (1) Amplitude varies as 8 – 10 V or 9 ± 1

(2) Two time period 100 μs (signal wave) and 8 μs (carrier wave)

Hence signal is $\left[3 \pm 1 \sin\left(\frac{2\pi t}{T_1}\right)\right] \sin\left(\frac{2\pi t}{T_2}\right)$

$= 9 \pm 1 \sin (2\pi \times 10^4 t) \sin 2.5\pi \times 10^5 t$

83. In a process, temperature and volume of one mole of an ideal monoatomic gas are varies according to the relation VT = K, where K is a constant. In this process the temperature of the gas is increased by ΔT. The amount of heat absorbed by gas is (R is a gas constant):

(1*) $\frac{1}{2}R\Delta T$ (2) $\frac{1}{2}KR\Delta T$ (3) $\frac{3}{2}KR\Delta T$ (4) $\frac{2K}{3}\Delta T$

Sol. VT = K

$\Rightarrow V\left(\frac{PV}{nR}\right) = k$

$\Rightarrow PV^2 = K$

$\because \; C = \frac{R}{1-x} + C_v$ (For polytropic process)

$C = \frac{R}{1-2} + \frac{3R}{2} = \frac{R}{2}$

$\therefore \; \Delta Q = nC \, \Delta T$

84. When 100 g of a liquid A at 100ºC is added to 50 g of a liquid B at temperature 75ºC, the temperature of the mixture becomes 90ºC. The temperature of the mixture, if 100g of liquid A at 100ºC is added to 50 g of liquid B at 50ºC, will be

(1) 85ºC (2) 60ºC (3*) 80ºC (4) 70ºC

Sol. $100 \times S_A \times [100 - 90] = 50 \times S_B \times (90 - 75)$

$2S_A = 1.5 \, S_B$

$S_A = \frac{3}{4}S_B$

Now, 100 × S_A × [100 –T] = 50 × S_B (T – 50)

$2\times\left(\frac{3}{4}\right)(100-T)=(T-50)$

300 – 3T = 2T – 100

400 = 5T

T = 80

85. In a hydrogen like atom, when an electron jumps from the M-shell to the L-shell, the wavelength of the emitted radiation is λ. If an electron jumps from N-shell to the L-shell, the wavelength of emitted radiation will be

(1) $\frac{27}{20}\lambda$ (2) $\frac{16}{25}\lambda$ (3) $\frac{25}{16}\lambda$ (4*) $\frac{20}{27}\lambda$

Sol. For M $\rightarrow$ L steel

$$\frac{1}{\lambda}=K\left(\frac{1}{2^2}-\frac{1}{3^2}\right)=\frac{K\times5}{36}$$

For N $\rightarrow$ L

$$\frac{1}{\lambda'}=K\left(\frac{1}{2^2}-\frac{1}{4^2}\right)=\frac{K\times3}{16}$$

$$\lambda'=\frac{20}{27}\lambda$$

86. A monochromatic light is incident at a certain angle on an equilateral triangular prism and suffers minimum deviation. If the refractive index of the material of the prism is $\sqrt{3}$, then the angle of incidence is

(1) 90° (2) 30° (3*) 60° (4) 45°

Sol. i = e

$r_1 = r_2 = \frac{A}{2} = 30°$

By Snell's law

$1\times \sin i=\sqrt{3}\times\frac{1}{2}=\frac{\sqrt{3}}{2}$

i = 60

87. In a double slit experiment, green light (5303Å) falls on a double slit having a separation of 19.44 μm and a width of 4.05μm.

(1) 10 (2*) 05 (3) 04 (4) 09

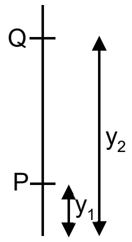
Sol. For diffraction

Location of 1st minima

$$y_1 = \frac{D\lambda}{a} = 0.2469D\lambda$$

Location of 2nd minima

$$y_2 = \frac{2D\lambda}{a} = 0.4938D\lambda$$



Now for interference

Path for interference

Path difference at P.

$$\frac{dy}{D} = 4.8\lambda$$

Path difference at P.

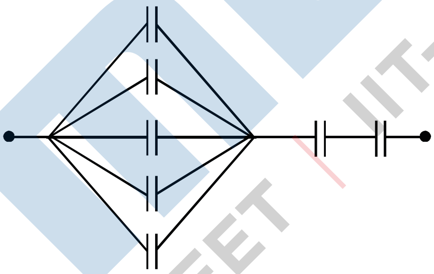
$$\frac{dy}{D} = 9.6\lambda$$

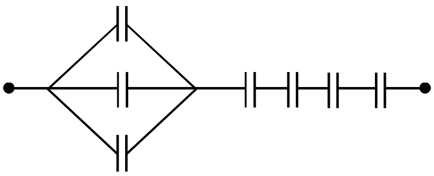
So orders of maxima in between P and Q is

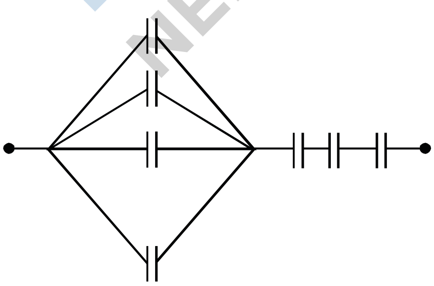
5, 6, 7, 8, 9

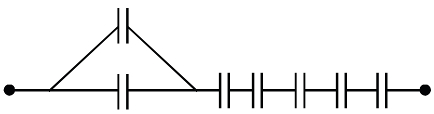
So 5 bright fringes all present between P & Q.

88. Seven capacitors, each of capacitance 2μF, are to be connected in a configurationto obtain an effective capacitance of $\left(\frac{6}{13}\right)$ μF. Which of the combinations, shown in figures below, will achieve the desired value?

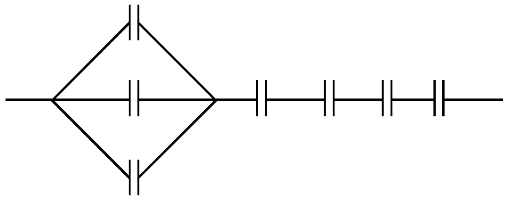
(1) 

(2*) 

(3) 

(4) 

Sol. $C_{aq} = \frac{6}{13}\mu F$



Therefore three capacitors most be in parallel to get 6 in

$$\frac{1}{C_{eq}} = \frac{1}{3C} + \frac{1}{C} + \frac{1}{C} + \frac{1}{C} + \frac{1}{C}$$

$$C_{eq} == \frac{3C}{13} = \frac{6}{13}\mu F$$

89. A particle of mass m and charge q is in an electric and magnetic field given by

$$\vec{E} = 2\hat{i} + 3\hat{j}\,;\ \vec{B} = 4\hat{j} + 6\hat{k}$$

The charged particle is shifted from the origin to the point P(x = 1, y = 1) along a straight path. The magnitude of the total work done is:

(1) (0.35) q (2*) 5 q (3) (2.5) q (4) (0.15)q

Sol. $\vec{F}_{net} = d\vec{E} + q\left(\vec{v}\times\vec{B}\right) = \left(2q\hat{i} + 3q\hat{i}\right) + q\left(\vec{v}\times\vec{B}\right)$

$W = \vec{F}_{net}\cdot\vec{S}$

$= 2q + 3q = 5q$

90. In a photoelectric experiment, the wavelength of the light incident on a metal is changed from 300 nm to 400 nm. The decrease in the stopping potential is close to : $\left(\frac{hc}{e} = 1240\text{ nm} - V\right)$

(1) 0.5 V (2)S 1.5 V (3*) 1.0 V (4) 2.0 V

Sol. $\frac{hc}{\lambda_1} = \phi + eV_1$ ......(i)

$\frac{hc}{\lambda_2} = \phi + eV_2$ ......(ii)

(i) – (ii)

$$hc\left(\frac{1}{\lambda_1} - \frac{1}{\lambda_2}\right) = e\left(v_1 - V_2\right)$$

$$\Rightarrow V_1 - V_2 = \frac{hc}{e}\left(\frac{\lambda_2 - \lambda_1}{\lambda_1 - \lambda_2}\right)$$

$$= (1240\text{ nm V})\frac{100\text{ nm}}{300\text{ nm}\times 400\text{ nm}}$$

$$= \frac{12.4}{12} \approx 1\text{ V}.$$